Quantum interferometry with and without an external phase reference

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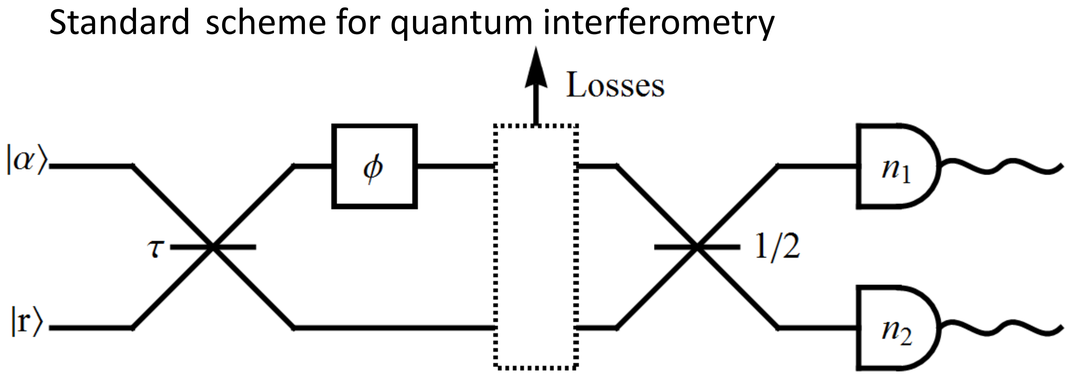
Introduction

Current interferometric setups are based on states without fixed photon number (only averaged photon number is fixed), e.g. coherent $|\alpha\rangle$ and squeezed vacuum state $|r\rangle$. However, if one considers general measurement scheme, this description assumes an additional phase reference which means one has to have access to reference beam. This is the reason of some misunderstandings in current literature, which we explain by considering four different cases of phase delays in Mach-Zehnder interferometer. We show that this is in general two parameter problem and full Fisher information matrix should be considered.

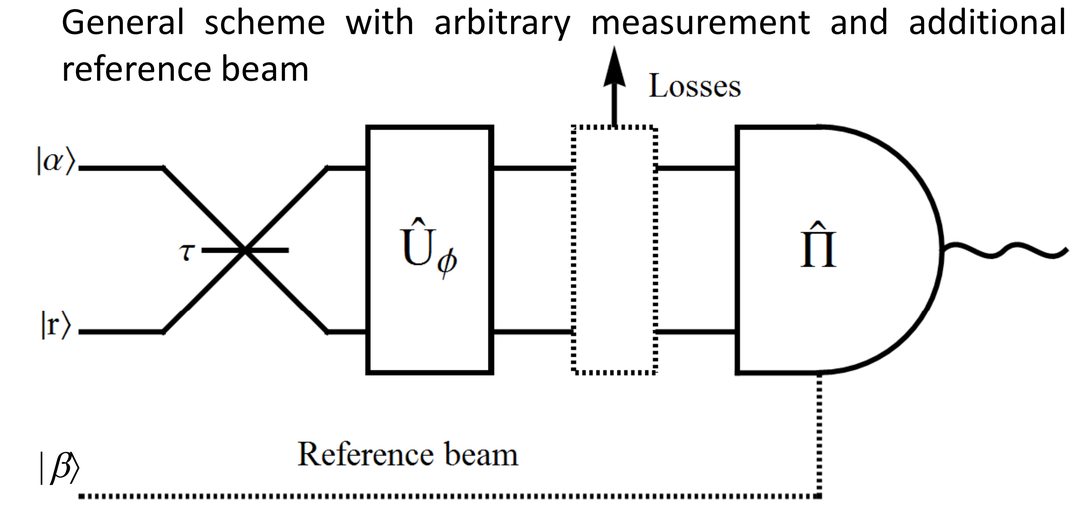
Schemes for interferometry

Standard scheme for quantum interferometry



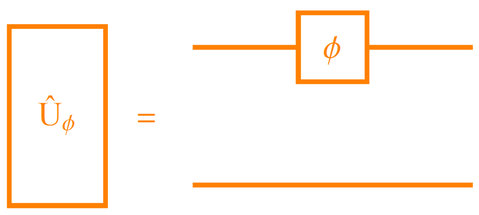
$$F_{cl} = \sum_{n_1,n_2} \frac{1}{p(n_1,n_2|\phi)} \left(\frac{dp(n_1,n_2|\phi)}{d\phi} \right)^2, \quad \Delta\phi_{cl} \geq \frac{1}{\sqrt{F_{cl}}}$$

General scheme with arbitrary measurement and additional reference beam



Three cases of phase delay

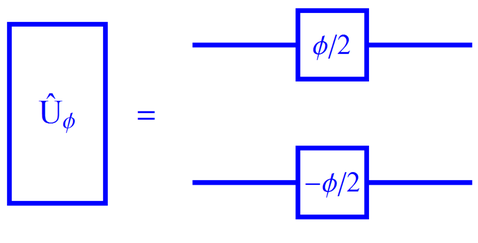
1.



Phase delay in one arm

$$F_1 = \mathrm{Tr}\left(\hat{\rho}\hat{\Lambda}^2\right), \quad \Delta\phi_1 = \frac{1}{\sqrt{F_1}}$$

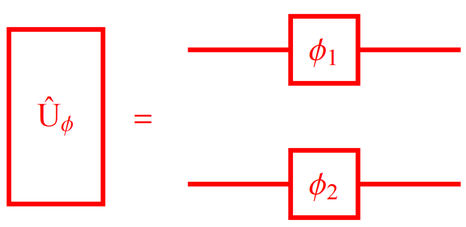
2.



Opposite phase delays in both arms

$$F_2 = \mathrm{Tr}\left(\hat{\rho}\hat{\Lambda}^2\right), \quad \Delta\phi_2 \geq \frac{1}{\sqrt{F_2}}$$

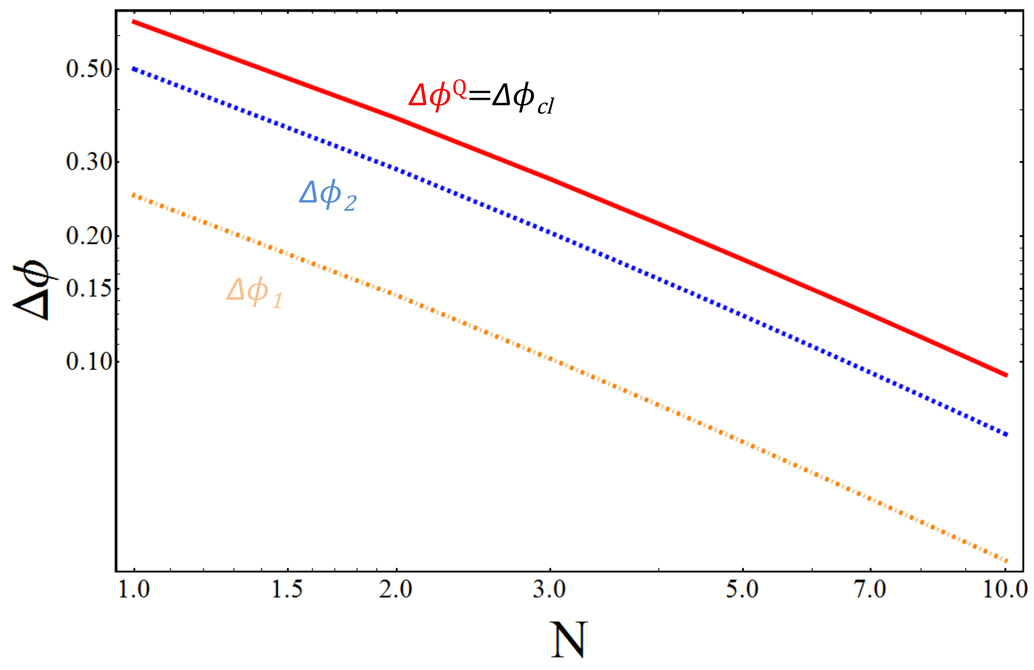
3.



Different phase delays in both arms

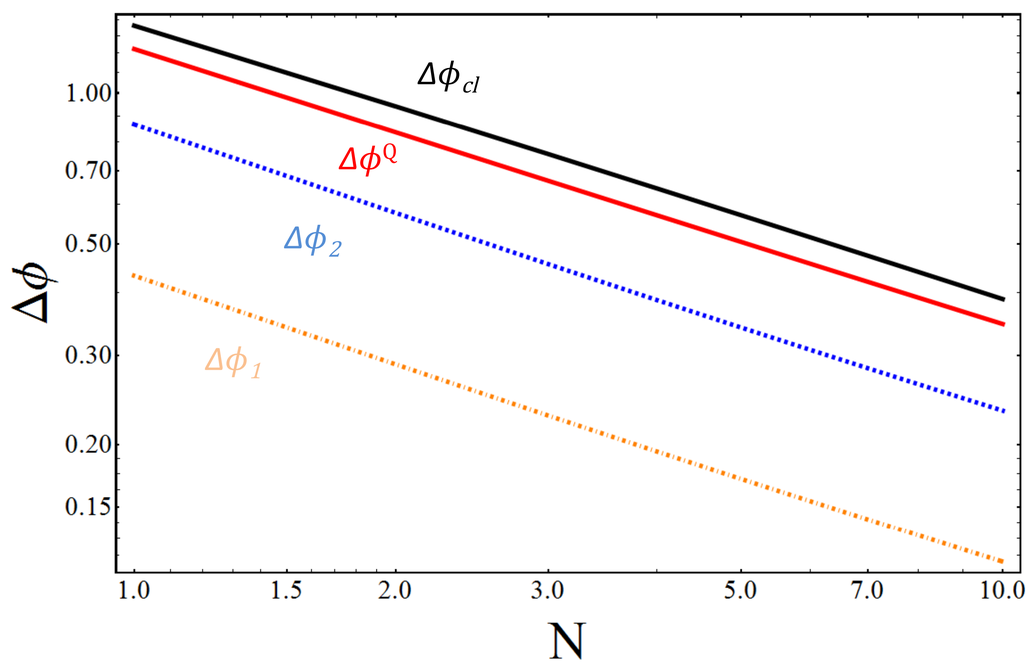
$$\mathcal{F} = \begin{pmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{pmatrix}, \quad F_{ij} = \frac{1}{2}\mathrm{Tr}\left(\hat{\rho}\hat{\Lambda}_i\hat{\Lambda}_j + \hat{\rho}\hat{\Lambda}_j\hat{\Lambda}_i\right), \quad \Delta\phi^Q_i \geq \left(\mathcal{F}^{-1}\right)_{ii}$$

Results: whithout losses



Best precision for optimal τ and without losses as a function of total photon number. In such case $\Delta\phi_Q = \Delta\phi_{cl}$. For $\Delta\phi_1$, $\Delta\phi_2$, $\tau = 0$, for $\Delta\phi_Q$, $\tau = 0.5$.

Results: losses



Best precision for optimal τ and with losses as a function of total photon number. In such case $\Delta\phi_Q \neq \Delta\phi_{cl}$. For $\Delta\phi_1$, $\Delta\phi_2$, $\tau = 0$, for $\Delta\phi_Q$, $\Delta\phi_{cl}$, $\tau = 0.5$.

Whithout reference beam

All three cases give different precisions only in the presence of reference beam. In other case state changes from $|\alpha\rangle|r\rangle$ to mixed state

$$\hat{\rho} = \int \frac{d\xi}{2\pi} \hat{U}_\xi |\alpha\rangle\langle\alpha| \otimes |r\rangle\langle r| U_\xi^+$$

which is averaged over phase, and one gets $\Delta\phi_1 = \Delta\phi_2 = \Delta\phi^Q = \Delta\phi_{cl}$.

Summary

In general, whole Fischer information matrix should be taken into consideration to obtain valid precision of phase estimation in the presence of reference beam. If not, different ways of placing phase delays result in different precisions. On the other hand, when the reference beam is absent, all cases give the same precision, which is equal to reference-beam case only when there are no losses. In the last situation, it can be obtained that the more photons reference beam contain, the better precision is achieved.

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